Friendly Workshop on Diophantine Equations and Related Problems 2019

The Booklet of Speakers and Abstracts

6-8 July, 2019

Bursa, TURKEY

FRIENDLY WORKSHOP ON DIOPHANTINE EQUATIONS AND RELATED PROBLEMS 2019

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Speakers and Abstracts

Edited by Ümit SARP

6-8 July, 2019

Bursa - TURKEY

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About Bursa

Bursa is a city in northwestern (called Marmara) of Türkiye and the seat of Bursa Province. The earliest known site at this location was Cius, which Philip V of Macedonia granted to the Bithynian king Prusias I in 202 BC, for his help against Pergamum and Heraclea Pontica (Karadeniz Ereğli). Prusias renamed the city after himself, as Prusa. It was later a major city, located on the westernmost end of the famous Silk Road, and was the capital of the Ottoman Empire following its capture from the shrinking Byzantine Empire in 1326. The capture of Edirne in 1365 brought that city to the force as well, but Bursa remained an important administrative and commercial center even after it lost its status as the sole capital. Shortly after it was taken by the Ottomans they developed a school of theology at Bursa. This school attracted Muslim scholars from throughout the Middle East and continued to function after the capital had been moved elsewhere. During the Ottoman rule, Bursa was the source of most royal silk products. Aside from the local production, it imported raw silk from Iran, and occasionally China, and was the factory for the kaftans, pillows, embroidery and other silk products for the royal palaces up through the 17th century. Another traditional occupation is knife making and, historically, horse carriage building. Nowadays one can still find hand-made knives as well as other products in rich variety produced by artisans, but instead of carriages, there are two big automobile factories FIAT and Renault.

The city is frequently cited as "Yeşil Bursa"(meaning Green Bursa) in a reference to the beautiful parks and gardens located across its urban tissue, as well as to the vast forests in rich variety that extend in its surrounding region. The city is synonymous with the mountain Uludağ (Olympus) which towers behind the city core and which is also a famous ski resort. The mausoleums of early Ottoman sultans are located in Bursa and the numerous edifices built throughout the Ottoman period constitute the city's main landmarks. The surrounding fertile plain, its thermal baths, several interesting museums, notably a rich museum of archaeology, and a rather orderly urban growth are further principal elements that complete Bursa's overall picture.

At present, there is a population of approximately 3 Million and it is Türkiye's fourth largest city, as well as one of the most industrialized and culturally charged metropolitan centers in the country. Karagöz and Hacivat shadow play characters were historic personalities who lived and are buried in Bursa. Bursa is also home to some of the most famous Turkish dishes, especially candied chestnuts and İskender kebap. Its peaches are also well-renowned. Among its depending district centers, Iznik (historic Nikea), is especially notable for its long history and important edices. Bursa is home to Bursa Uludağ University which is one of the high-scale universities in Türkiye and in the international area. It has 4.000 Academic staff and 79.000 students at different levels, 69.000 of them are undergraduates, at 15 Faculties. It has one of the highest numbers, around 5.000, of foreign students amongst 181 Turkish universities because of its ongoing relationships at international arena.

Preface

Dear Participants, on behalf of the Organizing Committe, we would like to welcome you all to Bursa and Bursa Uludağ University for our workshop.

Bursa Uludağ University hosting the workshop is a middle aged University in Turkish standards. Being 44 years old, it has almost completed the infrastructure and human resources. Mathematics Department is one of the oldest departments of the University which has been giving undergraduate and postgraduate education since 1983. Main research areas of the staff are Algebra, Applications of Mathematics and Differential Equations, Complex Analysis, Differential Geometry, Diophantine Equations, Discrete Group Theory, Elliptic Curves, Graph Theory, Number Theory, Projective Geometry.

The purpose of this workshop is to bring together people working on Diophantine equations and related problems. The idea was raised by Professors Gökhan Soydan (Bursa Uludağ University, Turkey) and Alain Togbé (Purdue University, Northwest, USA) during the 2017 Journées Arithmétiques of Caen (France). We would like this workshop to be a satellite conference to 2019 Journées Arithmétiques of Istanbul (Turkey).

So this workshop is an opportunity to anyone who would to present his/her research on classical effective methods in Diophantine number theory that include Baker's method, the hypergeometric method, reduction method of Baker-Davenport ... For decades, research on Diophantine equations has been governed by these famous methods. For example, one can cite the use of these methods to solve the famous conjecture that there is no Diophantine quintuple, the progress in solving the Diophantine equation $ax^p + by^q = cz^r$, numerous Diophantine problems related to recurrences. A second aim of this friendly workshop is to create a good atmosphere for collaboration.

Again, we warmly welcome you to Bursa and wish you a fruitful and pleasant workshop.

This workshop is partially supported by TÜBİTAK under Project No: 117F287. Organizing Committe would like to thank to TÜBİTAK for this financial support.

Organizing Committe

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Abstracts

On linear combinations of products of consecutive integers

András Bazsó

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Abstract: For $n = 0, 1, 2, \ldots$, let a_n be a sequence of rational numbers, and let

$$f_n^{a_n}(x) = \sum_{k=0}^{n} a_k \prod_{j=0}^{k} (x + j).$$

In the talk we study Diophantine equations involving the polynomials $f_n^{a_n}(x)$ for certain sequences a_n. The results presented are partly joint with Attila Bérczes, Lajos Hajdu and Florian Luca.

Keywords: Sums of products, blocks of consecutive integers, polynomial values.

2010 Mathematics Subject Classification: 11D41

Some Diophantine problems connected to binary recurrences

Attila Bérczes

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Abstract: Binary recurrence sequences are in the focus of research for a long time. In the frame of Diophantine number theory researchers were investigating divisibility properties of members of recurrence sequences and many more Diophantine properties of them. However, there are several Diophantine problems and Diophantine equations which are not directly connected to recurrences, but linear recurrences (and theorems proved about them) appear as tools in the solution of these problems.

In my talk I will present some results on Diophantine equations and problems containing linear recurrence sequences, and also some diophantine results, where the linear recurrences appear only in the proof of the results.

A Family of Thue Equations over Imaginary Quadratic Fields

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Abstract: Inspired by my academic grandfather's Thomas's work on solving a family of cubic Diophantine equations [6], my coauthors and I explore other families of equations. We let t be an imaginary quadratic integer and define a family of Thue equations over imaginary quadratic integers with t as a parameter. After a brief introduction, I will present some preliminary work on solving such a family of Thue equations. The proof uses methods and techniques that expand upon work as seen in [1]–[9].

Keywords: Thue equations, imaginary quadratic fields

2010 Mathematics Subject Classification: 11D25, 11D41, 11D45, 11D75

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Effective results for Diophantine equations over finitely generated domains

Kálmán Győry

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Abstract: In the first part of my talk, I give a brief survey of effective results and methods for Diophantine equations over finitely generated domains (FGD's) over $\mathbb{Z}$ which may contain transcendental elements, too. The first general results of this kind were obtained by Győry in 1983 for decomposable form equations and discriminant equations. In 2013, Evertse and Győry combined Győry's method with an effective result of Aschenbrenner from 2004 concerning ideal membership in polynomial rings over $\mathbb{Z}$ to establish effective finiteness theorems for unit equations over FGD's over $\mathbb{Z}$. Using their method, recently Bérczes, Evertse, Győry and Koymans proved such results for several important classes of polynomial and exponential Diophantine equations. In the second part of the talk I present some new effective results for equations of this type.

Keywords: Polynomial diophant equation, exponential diophantine equation, effective results, finitely generated ground domains.

2010 Mathematics Subject Classification: 11D53, 11D59, 11D61, 11D99

Skolem's conjecture confirmed for a family of exponential equations

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Abstract: Skolem's conjecture states that if an exponential Diophantine equation is not solvable, then it is not solvable modulo an appropriately chosen modulus. The conjecture has been proved only for rather special classes of equations. Schinzel [6] (extending results of Skolem [10]) proved the conjecture for equations of the form $\alpha_1^{x_1} \cdots \alpha_k^{x_k} = \beta$ where $\alpha_1, \ldots, \alpha_k$ and β are fixed elements of a number field, and $x_1, \ldots, x_k$ are unknown integers. Bartolome, Bilu and Luca [1] proved the conjecture for equations of the shape $\lambda_1\alpha_1^n + \cdots + \lambda_k\alpha_k^n = 0$, where $\lambda_1, \ldots, \lambda_k$, and $\alpha_1, \ldots, \alpha_k$ are elements of a number field K such that the multiplicative group generated by $\alpha_1, \ldots, \alpha_k$ is of order one, and n is a variable. We note that the results in [1] can be derived from those in [6, 7] - though not in a straightforward way [9]. For related results, see also [8, 4]. Beside these, several particular equations have been treated by methods based upon Skolem's principle; see e.g. [2, 3] and the references there.

In the talk we present a result showing that the conjecture is valid for the Catalan equation $u^x - v^y = 1$ provided that one of u, v is a prime. This is the first instance where the conjecture is proved for a family of equations with more than one terms on the left hand side, of which the bases are multiplicatively independent.

The new result are joint with **R. Tijdeman**, and are published in [5].

Keywords: Exponential Diophantine equations, Skolem's conjecture

2010 Mathematics Subject Classification: 11D61, 11D79

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Solutions of a diophantine equation with binomial coefficients and Lucas numbers

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Abstract: Let L_m denote the m^{th} Lucas number. In this paper, we show that the solutions to the diophantine equation

$$\binom{2^t}{k} = L_m \tag{1}$$

in non-negative integers t, $k \leq 2^{t-1}$ and m are

$$(t,k,m) = (1,1,0),\ (2,1,3) \text{ and } (a,0,1)$$

with non-negative integers a.

Keywords: Lucas number, binomial coefficient, diophantine equation.

2010 Mathematics Subject Classification: 11B39, 11D72

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Perfect Powers With All Equal Digits But One

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Abstract: We show that for any fixed integer $l \geq 3$, there are only finitely many perfect l-th powers all of whose digits are equal but one, except for two trivial families, namely 10^{ln} for $l \geq 3$ and $8 \cdot 10^{3n}$ for $l = 3$. We also discuss some related open problems.

Keywords: Perfect powers, digits.

2010 Mathematics Subject Classification: Primary 11D75; Secondary 11J75.

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A note on the ternary purely exponential Diophantine equation $A^x + B^y = C^z$ with $A + B = C^2$

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Abstract: Let A, B, C be coprime positive integers with $\min\{A, B, C\} > 1$. Consider the equation

$$A^x + B^y = C^z, \quad x, y, z \in \mathbb{N} \tag{2}$$

for some triples (A, B, C) with $A+B = C^2$. Recently, many authors studied the solutions (x, y, z) of the equation (2) (see [1, 3, 4], [7]-[11]). In 2017, N. Terai and T. Hibino [11] proved that if

$$A = 3pm^2 - 1,\ B = (p-3)pm^2 + 1,\ C = pm, \tag{3}$$

where p is an odd prime with $p < 3784$ and $p \equiv 1 \pmod 4$, m is a positive integer with $3 \nmid m$ and $m \equiv 1 \pmod 4$, then the equation (3) has only the solution $(x, y, z) = (1, 1, 2)$.

Let ℓ, m, r be positive integers such that

$$2 \nmid \ell,\ 3 \nmid \ell m,\ \ell > r,\ 3 \mid r. \tag{4}$$

In this work, we consider the equation (2) for the case

$$A = r\ell m^2 - 1,\ B = (\ell - r)\ell m^2 + 1,\ C = \ell m. \tag{5}$$

We prove that if $\min\{r\ell m^2 - 1, (l - r)\ell m^2 + 1\} > 30$, then the equation (2) has only the solution $(x, y, z) = (1, 1, 2)$ where A, B, C satisfy (5) with (4). And also we prove that if $p \geq 11$ and $3 \nmid m$, then (2) has only the solution $(x, y, z) = (1, 1, 2)$ where A, B, C satisfy (3). On the proofs, we use the BHV theorem on the existence of primitive divisors of Lehmer numbers due to Y. Bilu, G. Hanrot and P. M. Voutier [2].

The first and third authors were supported by TÜBİTAK (the Scientific and Technological Research Council of Turkey) under Project No: 117F287.

Keywords: Ternary purely exponential Diophantine equation, BHV theorem on the existence of primitive divisors of Lehmer numbers.

2010 Mathematics Subject Classification: 11D61

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Terms of binary recurrence sequences which are products of factorials

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Abstract: A conjecture of Hickerson states that the equation $n! = a_1!a_2! \cdots a_k!$ with $2 \leq a_k \leq a_{k-1} \leq \cdots \leq a_2 \leq a_1 \leq n-2$ in positive integers implies $n \leq 16$. This is open. For a binary recurrence sequence $\{U_n\}_{n \geq 0}$, we show that the largest n for which $| U_n |= m_1!m_2! \cdots m_k!$ with $1 < m_1 \leq m_2 \leq \cdots \leq m_k$ satisfies $n < 3 \times 10^5$. We also give better bounds in case the roots of the binary recurrence sequence are real. As a consequence, we show that if $\{X_k\}_{k \geq 1}$ is the sequence of X-coordinates of a Pell equation $X^2 - dY^2 = \pm 1$ with a nonsquare integer $d > 1$, then the equation $X_k = n!$ implies $k = 1$. This is a joint work with F. Luca and M. Sias.

Keywords: Lucas Sequences, Factorials, Pell equation

2010 Mathematics Subject Classification: 11B37, 11D61

Evaluation of character sums and arithmetic questions

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Abstract: Characters sums have been evaluated in connection with counting rational points on algebraic curves over finite fields. In this presentation, we survey how these sums can be used to obtain global information about elliptic curves. In particular, Nagao's Conjecture provides a tight link between ranks of elliptic surfaces over the rational field and the aforementioned sums. If time allows, we will display how character sums may be used to shed some light on other arithmetic questions.

Keywords: Character sums, elliptic surfaces, Nagao's Conjecture.

2010 Mathematics Subject Classification: 11G05; 11T24; 14H52

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On extensibility of some parametric families of $D(-1)$-pairs to quadruples in rings of integers of the imaginary quadratic fields

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Abstract: Let R be a commutative ring. A set of m distinct elements in R such that the product of any two distinct elements increased by $z \in R$ is a perfect square is called a $D(z)$-m-tuple in R.

Let $z = -1$, $R = \mathbb{Z}[\sqrt{-t}], t > 0$ and p be an odd prime number. We study the extendibility of $D(-1)$-pairs $\{1, p\}$ and $\{1, 2p^i\}, i \in \mathbb{N}$ to quadruples in R. To do it, we study the equation $x^2 - (p^{2k+2}+1)y^2 = -p^{2l+1}$, $l \in \{0, 1, \ldots, k\}, k \geq 0$ and prove that it is not solvable in positive integers x and y.

Keywords: Diophantine triple, quadratic field, Diophantine equation, Diophantine quadruple.

2010 Mathematics Subject Classification: 11D09;11R11

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The Shuffle Variant of a Diophantine equation of Miyazaki and Togbé

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Abstract: Suppose that a and b are odd positive integers. In 2012, T. Miyazaki and A. Togbé, [3], gave all of the solutions of the Diophantine equations $(2am-1)^x+(2m)^y=(2am+1)^z$ and $b^x+2^y=(b+2)^z$ with $a>1$ and $b\geq 5$ in positive integers. In this work, we propose a similar problem (which we call the shuffle variant of a Diophantine equation of Miyazaki and Togbé). Here we first prove that the Diophantine equation $(2am+1)^x+(2m)^y=(2am-1)^z$ has only the solution $(a,m,x,y,z)=(2,1,2,1,3)$ with any fixed positive integer $a>1$ in positive integers. Then using this result, we show that the Diophantine equation $b^x+2^y=(b-2)^z$ has no solutions with any fixed odd positive integer $b\geq 7$ in positive integers. On the proofs, we use elementary methods and Laurent's refinement of Baker's theorem.

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Keywords: Exponential Diophantine equation, Baker's method.

2010 Mathematics Subject Classification: 11D61, 11J86.

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New results on the Jeśmanowicz conjecture

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Abstract: The Jeśmanowicz conjecture states that if $(p^2-q^2)^2+(2pq)^2=(p^2+q^2)^2$, then the only solution to the diophantine equation $(p^2-q^2)^x+(2pq)^y=(p^2+q^2)^z$, is $x=y=z=2$. Several partial results exist for this conjecture. We will provide new results using the theory of class numbers of binary quadratic forms. For instance, we provide an infinite family of values for which there are no solutions in the case when $4 \,||\, pq$.

Keywords: Exponential Diophantine equation, Jeśmanowicz' conjecture, class number of quadratic fields.

2010 Mathematics Subject Classification: 11E16, 11R29, 11D61.

On the Exponential Diophantine Equation $(a^n-2)(b^n-2)=x^2$

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Abstract: In this paper, we deal with the equation

$$(a^n-2)(b^n-2)=x^2, 2 \leq a < b, a, b, x, n \in N. \tag{6}$$

We solve the equation (6) for $(a,b) \in \{(2,10),(4,100),(10,58),(3,45)\}$. Moreover, we show that $(a^n-2)(b^n-2)=x^2$ has no solution n,x if $2 \mid n$ and $\gcd(a,b)=1$. We also give a conjecture which says that the equation $(2^n-2)((2P_k)^n-2)=x^2$ has only the solution $(n,x)=(2,Q_k)$, where $k>3$ is odd and P_k, Q_k are Pell and Pell Lucas numbers, respectively. We also conjecture that if the equation $(a^n-2)(b^n-2)=x^2$ has a solution n,x, then $n \leq 6$.

Keywords: Pell equation, exponential Diophantine equation, Lucas sequence.

2010 Mathematics Subject Classification: 11D61;11D31;11B39

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On a family of biquadratic fields that do not admit a unit power integral basis

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Abstract: In this talk, we will answer the following question: For which number fields does there exist a power integral basis consisting of units? In fact, we consider the following family of biquadratic fields

$$\mathbb{K} = \mathbb{Q}(\sqrt{18n^2 + 17n + 4}, \sqrt{2n^2 + n})$$

and prove that if $18n^2 + 17n + 4$ and $2n^2 + n$ are both square-free, then $\mathcal{K}$ does not admit a power integral basis consisting of units.

Keywords: Unit sum number problem, Power integral bases, System of Pell equations.

2010 Mathematics Subject Classification: 11R16,11D57,11R33

On Some Pure Ternary Exponential Diophantine Equations

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Abstract: We generalize the conjectures of Jeśmanowicz and Terai-Cao-Le about ternary pure exponential Diophantine equation $a^x + b^y = c^z$ with variables in the ring of rational integers and with fixed positive integers $a, b, c \geq 2$ whether coprime or not coprime. We list all negative integer solutions of equation $(an)^x +$

$(bn)^y = (cn)^z$ with $a, b, c > 0$ and $\gcd(a, b, c) = 1$. We give all integer solutions of a family of equations to verify the conjecture of Yuan-Han.

Keywords: Pure Ternary Exponential Diophantine Equations, Jeśmanowicz-Terai-Cao-Le Conjecture, Yuan-Han Conjecture, P-adic Anynasis.

2010 Mathematics Subject Classification: 16D61

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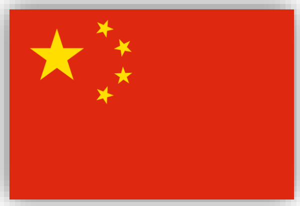

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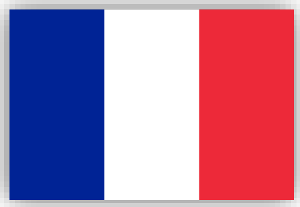

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Mathematics is mainly a matter of patience.
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